Limit theorems for functionals of stationary random fields under integrability conditions on spectral densities

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One of the classical methods to derive central limit theorems for various functionals of stationary random fields consists in computing the cumulants of the corresponding functional and evaluating their asymptotic behaviour.

Taking consideration of the cumulants in the spectral domain one is lead to deal with some kind of convolutions of spectral densities of a random field with particular kernel functions.

To state the central limit theorems, we develop the approach for estimation of this kind of convolutions which is based on the generalized Hölder inequality, so-called Hölder-Young-Brascamp-Lieb inequality. Under prescribed conditions on the integrability indices for a set of functions $f_i \in L_p(S,d\mu)$, $i = 1, \ldots, n$, this inequality allows to write upper bounds for the integrals of the form

$$\int_{S^n} \prod_{i=1}^n f_i(l_i(x_1, \ldots, x_m)) \prod_{j=1}^m \mu(dx_j)$$

with $l_i : S^m \to S$ being linear functionals. An even more powerful tool, the nonhomogeneous Hölder-Young-Brascamp-Lieb inequality, covers the case when the above functions $f_i$ are defined over the spaces of different dimensions: $f_i : S^{n_i} \to \mathbb{R}$.

In such a way we are able to state limit theorems for functionals of stationary random fields under the conditions of integrability of spectral densities.

We consider in particular linear functionals and bilinear forms of random fields, and some other functionals, which are important for statistical estimation of random fields in nonparametric and parametric settings.

The talk is partly based on the results obtained jointly with N. Leonenko and F. Avram.

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