On the dichotomy of a locally compact semitopological monoid of order isomorphisms of principal filters of a power of the positive integers with adjoined zero

Taras Mokrytskyi
National University of Lviv, Lviv, Ukraine

For an arbitrary positive integer $n$ by $(\mathbb{N}^n, \leq)$ we denote the $n$-th power of the set of positive integers $\mathbb{N}$ with the product order:

$$(x_1, \ldots, x_n) \leq (y_1, \ldots, y_n) \quad \text{if and only if} \quad x_i \leq y_i \quad \text{for all} \quad i = 1, \ldots, n.$$ 

It is obvious that the set of all order isomorphisms between principal filters of the poset $(\mathbb{N}^n, \leq)$ with the operation of composition of partial maps form a semigroup. This semigroup will be denoted by $\mathcal{IPF}(\mathbb{N}^n)$. The semigroup $\mathcal{IPF}(\mathbb{N}^n)$ is some generalization of the bicyclic semigroup $\mathcal{C}(p,q)$. Hence it is natural to ask: what algebraic and topological properties of the semigroup $\mathcal{IPF}(\mathbb{N}^n)$ are similar to ones of the bicyclic monoid? The structure of the semigroup is studied in [4]. There we showed that $\mathcal{IPF}(\mathbb{N}^n)$ is a bisimple, $E$-unitary, $F$-inverse monoid, describe Green’s relations on $\mathcal{IPF}(\mathbb{N}^n)$ and its maximal subgroups. We proved that $\mathcal{IPF}(\mathbb{N}^n)$ is isomorphic to the semidirect product of the direct $n$-th power of the bicyclic monoid $\mathcal{C}(p,q)$ by the group of permutation $S_n$, every non-identity congruence on $\mathcal{IPF}(\mathbb{N}^n)$ is group and describe the least group congruence on $\mathcal{IPF}(\mathbb{N}^n)$. We showed that every Hausdorff shift-continuous topology on $\mathcal{IPF}(\mathbb{N}^n)$ is discrete and discuss embedding of the semigroup $\mathcal{IPF}(\mathbb{N}^n)$ into compact-like topological semigroups.

In the paper [2] was proved the following dichotomy: a locally compact semitopological bicyclic semigroup with an adjoined zero is either compact or discrete. The above dichotomy was extended to locally compact $\lambda$-polycyclic semitopological monoids in [1] and to locally compact semitopological interassociates of the bicyclic monoid in [3].

By $\mathcal{IPF}(\mathbb{N}^n)^0$ we denote the monoid $\mathcal{IPF}(\mathbb{N}^n)$ with adjoined zero.

We extend the above results and prove the following theorem

**Theorem.** Every Hausdorff locally compact shift-continuous topology on $\mathcal{IPF}(\mathbb{N}^n)^0$ is either compact or discrete.

**Corollary.** Every Hausdorff locally compact semigroup topology on $\mathcal{IPF}(\mathbb{N}^n)^0$ is discrete.

**References**


e-mail: tmokrytskyi@gmail.com